Torsion of Thin-Walled Bars[1]

Review of Circular Shafts

The shear stress for a circular cross section varies linearly. Figs. 1 and 2 show the directions and magnitudes of the shear stresses for solid and annular cross sections.

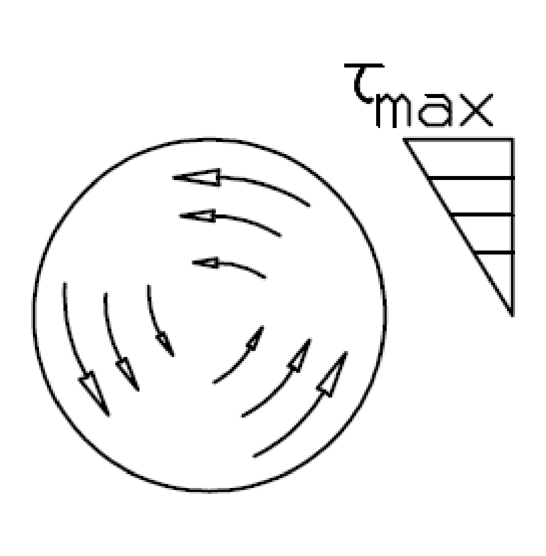


Fig.1 Solid round bar.

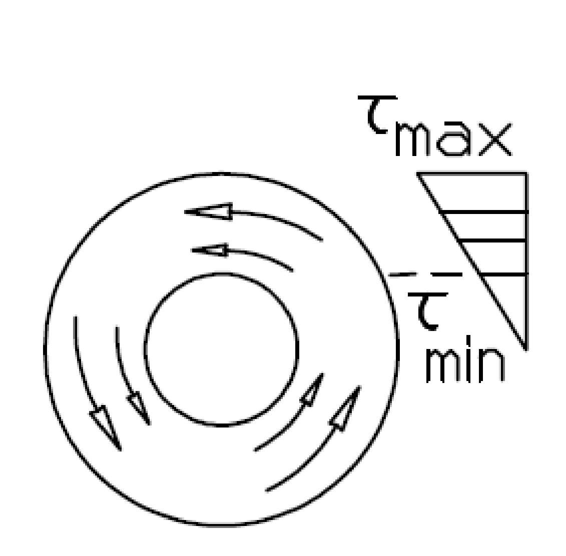


Fig. 2 Annular round bar.

The formulas for calculating the shear stresses and the angle of twist are:

$$\tau = \frac{Tr}{J} \quad ; \quad \tau_{\max} = \frac{TR_0}{J} \quad ; \quad \phi = \frac{TL}{GJ}$$

The polar second moment of area J = π / 2 [$R_o^2 - R_i^2$]

For thin walled tubes with t << $R_m = (R_o + R_i)/2$,

$$\frac{\tau_{\min}}{\tau_{\max}} = \frac{R_i}{R_0} \approx 1$$

An Approximate Formula for Thin-walled Circular Tubes

Fig. 3 shows a thin-walled cylindrical bar subjected to a twisting moment T. The mean radius is R_m. The variation of the shear stress with the thickness is neglected. A typical small area of length $R_m\, d\theta$ and thickness "t" transmits an increment of force $dF = \tau\, t\, R_m\, d\theta$. The moment of the incremental forces about the axis of the cylinder equals the applied torque T.

$$\therefore\ T = \oint R_m dF = \int_0^{2\pi} R_m \tau\ t R_m d\theta = \tau\ t R_m^2 2\pi = 2tA_0\tau$$

Where, $A_0 = \pi R_m^2$ is the **area enclosed by the median line**. It is not the area of the cross section material. Then

[1]Ahmad Mansour

$$\therefore \tau = \frac{T}{2tA_0}$$

This formula is valid for thin-walled non-circular tubular cross sections as will be disused later.

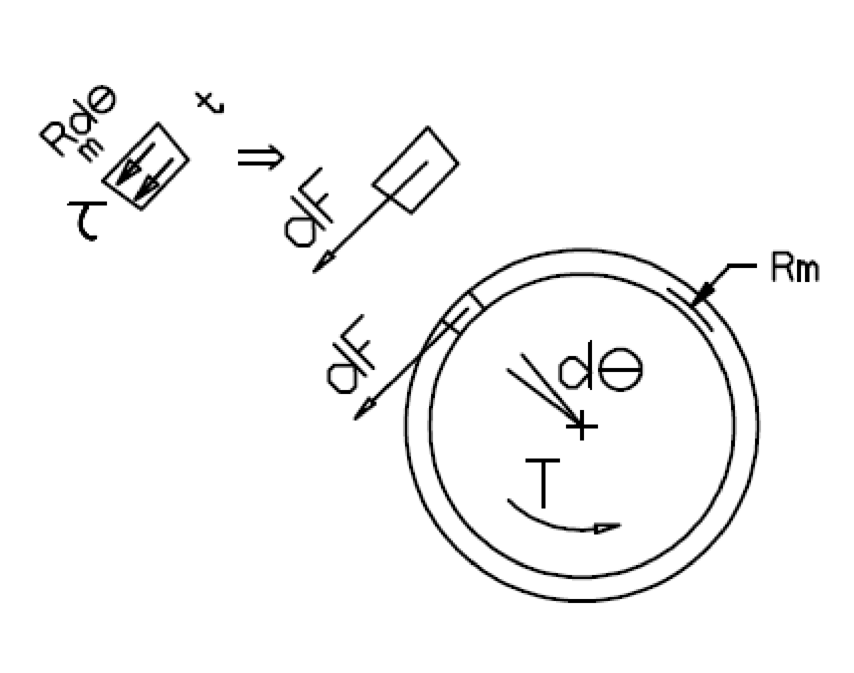


Fig. 3 Approximate formula for thin tubes. $dF = \tau\,[R_m d\theta\ t]$

Example 1

For a thin tube, calculate the percentage error in the approximate formula for

$\lambda \equiv t / R_0 =$	0.05	0.1	0.15	0.2

Take R_0 = 10 mm and T = 10 N.m. State whether the formula is conservative.

Solution:

Exact formula:

$R_i = R_0 - t = (1- \lambda)\, R_0$

$J = \pi/2\, (R_0^4 - R_i^4) = \pi/2\, R_0^4\, (1 - (1- \lambda)^4)$

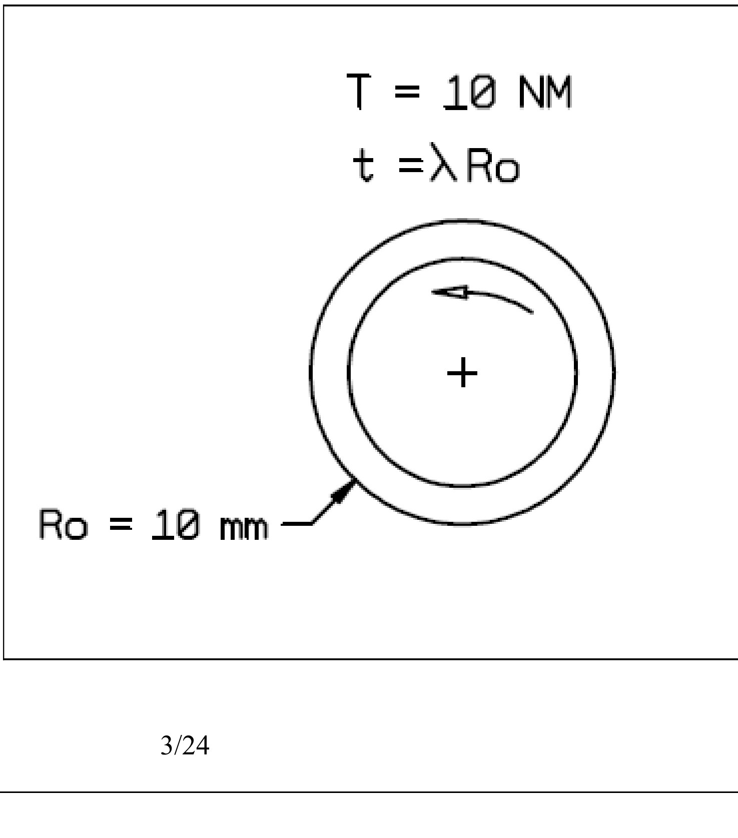
$\tau_{max} = (T\, R_0) / J = 6366197.7 / (1 - (1- \lambda)^4)$

Approximate formula:

$R_m = (R_0 + R_i) / 2 = R_0\, (1- 0.5\, \lambda)$

$A_0 = \pi\, R_m^2 = \pi\, R_0^2\, (1-0.5\, \lambda)^2$

$\tau_{max\text{-}appr} = 1591549.4 / (\lambda\, (1 - 0.5\, \lambda)^2)$



$\lambda \equiv t / R0$ =	0.05	0.1	0.15	0.2
$\tau_{max-exact}$ MPa	34.32	18.512	13.319	10.7829
$\tau_{max-appr}$ MPa	33.484	17.635	12. 401	9.824
Error %	-2.4 %	-4.7 %	-6.9 %	-8.9 %

Where, error % = $(\tau_{max-appr} - \tau_{max-exact}) / \tau_{max-exact}$ x 100%

The approximate formula predicts lower stresses than the exact values. For example, the error is about 5 % at λ = 0.1. The error is on the unsafe side (why?).

Types of Thin-Walled Bars

(1) Thin-walled tubular bars where the shear stress is constant across the thickness, Figs. 4 (a). They are also known as closed thin-walled cross sections.

(2-a) Rectangular, and open cross sections of uniform thickness, Figs. 4 (b). The shear stress varies its direction and magnitude across the thickness.

(2-b) Branched open cross sections and open cross sections with variable thickness, Figs. 4 (c). The shear stress varies its direction and magnitude across the thickness.

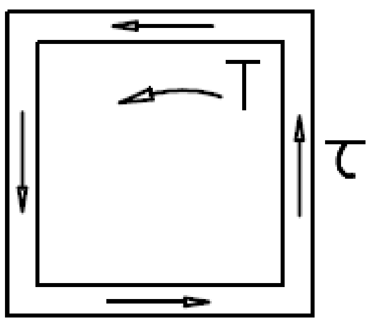


Fig. 4 (a) ↕

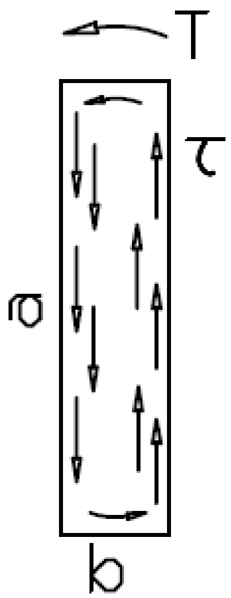


Fig. 4 (b) ↕

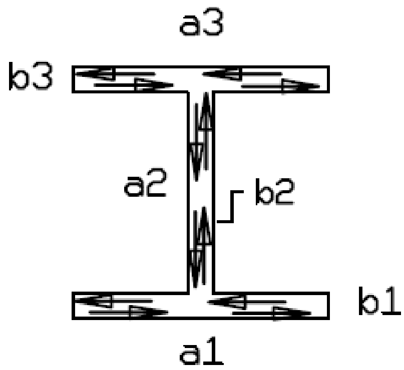


Fig. 4 (c) ↕ T is CCW

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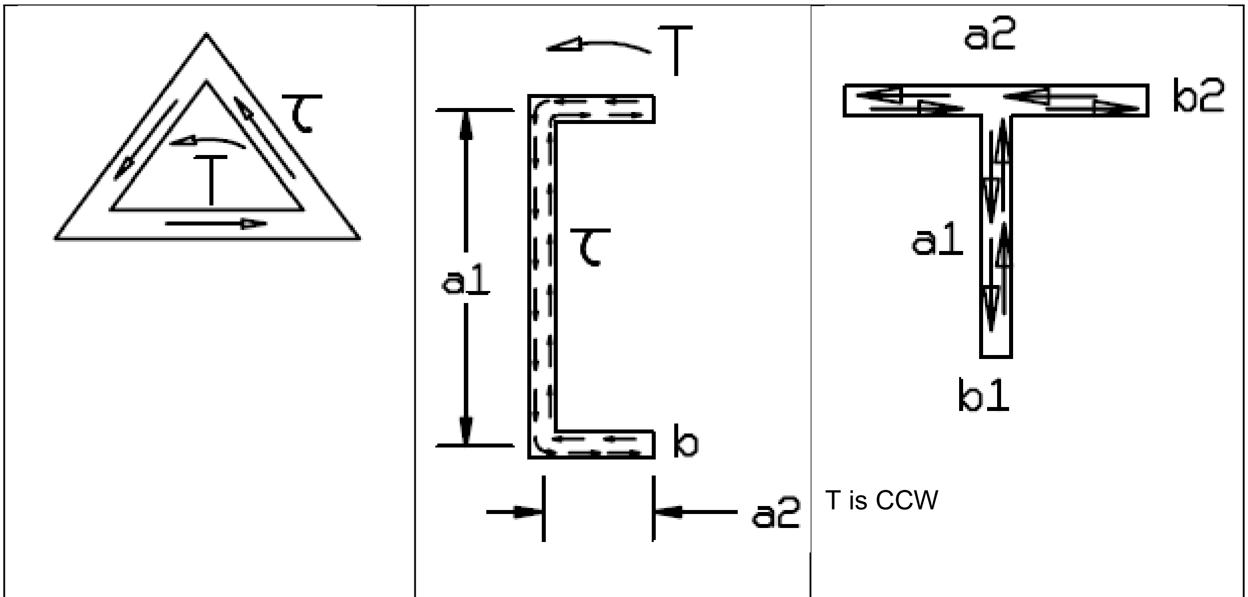


Fig. 4 (a)	Fig. 4 (b)	Fig. 4 (c)

The stresses developed in tubular cross sections are much less than those developed in open cross sections. This is because the stresses in tubular cross sections resist the applied torque with a large resisting arm, Fig. 5.

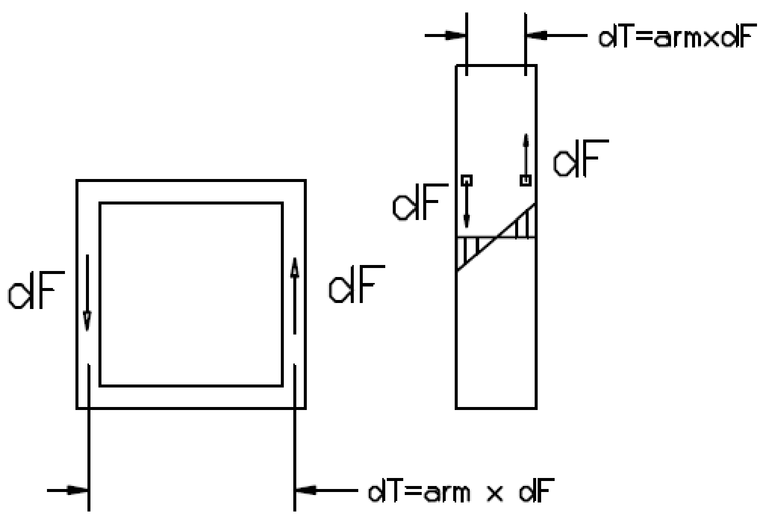


Fig. 5 Shear stresses in closed sections are smaller than in open sections.

Non-Circular Tubes of Variable wall thickness

These cross sections are also known as closed thin-walled cross sections, Figs. 6 (a) and (b). For a tubular bar with variable thickness

$\tau\, t\,)_a = \tau\, t\,)_b$ = constant = q

and

$$\tau = \frac{T}{2tA_0}$$

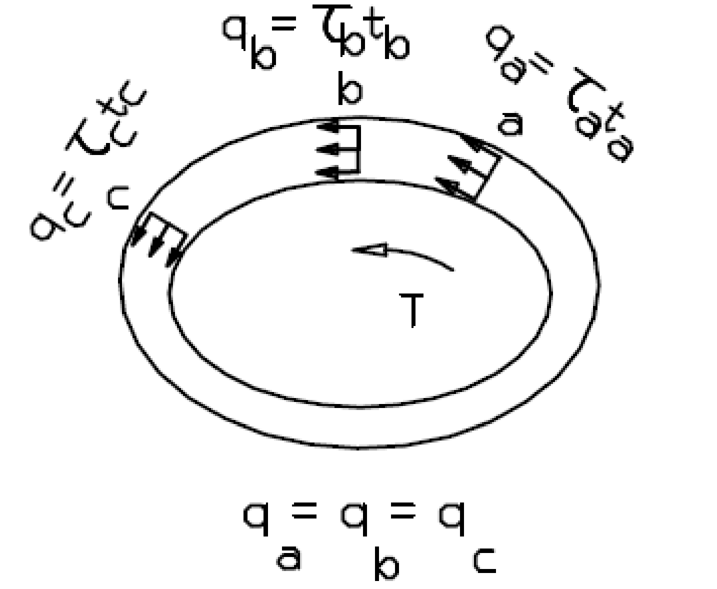


Fig. 6 (a) Tubular bar of variable thin-walled cross section. The shear flow q = τ t is constant.

Where, A_0 is the enclosed area by the median line. The shear stress τ varies inversely with t. Appendices I and II give proofs of these formulas.

The shear stress has a <u>maximum value at the minimum thickness</u>. The quantity "τ t" is the shear flow "q" because it resembles liquid flow in channels.

$\therefore$ q = τ t

The angle of twist is:

$$\tau = \frac{TL}{KG} \quad ; \quad K = \frac{4A_0^2}{\oint \frac{ds}{t}}$$

when t is constant

$$K = \frac{4A_0^2 t}{S}$$

Where S is the length of the closed loop median line, Fig. 6 (b).

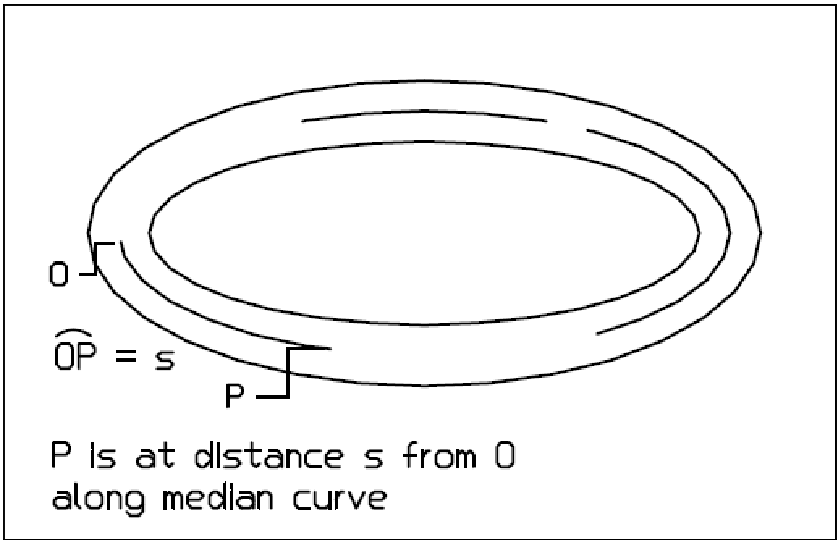
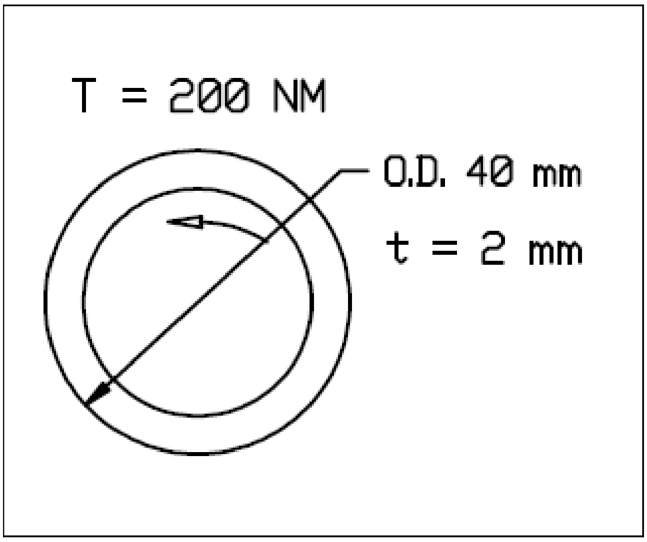


Fig. 6 (b) Definition of s; the distance measured along the median line from an arbitrary point O.

Example 2

Compare between Bredt's formula and the exact theory when used to evaluate the angle of twist of thin-walled circular tube. Take D_0 = 40 mm, t = 2 mm, G = 80 GPa, L = 1 m, and T = 200 Nm.



Solution:

Exact theory:

D_i = 40 -2 x 2 = 36 mm

J = π/32 ($0.04^4 - 0.036^4$) = 8.6431 x 10^{-8} m^4

Φ = (T L) / GJ = 0.0289 rad = 1.657°

Approximate formula:

R_m = 19 mm

$A_0 = \pi R_m^2 = 1.13411 \times 10^{-3}$ m^2

$$\oint \frac{ds}{t} = \frac{S}{t} = \frac{2\pi R_m}{t} = 59.69$$

$$K = \frac{4A_0^2}{\oint \frac{ds}{t}} = 8.6192 x 10^{-8} \quad m^4$$

$$\phi = \frac{TL}{GK} = \frac{200 x 1}{80 x 10^9 \; x 8.6192 x 10^{-8}} = 0.029 \quad rad = 1.66^{\circ}$$

The two results are very close and Φ ≈ 1.7°.

Example 3

A torque of 1 kNm acts on a bar with the shown cross section, Fig. 7 (a). Find the magnitude of the maximum shear stress and the angle of twist per unit length. Take t_{DC} = t_{AB} = 4 mm, t_{AD} = t_{BC} = 6 mm, and G = 80 GPa.

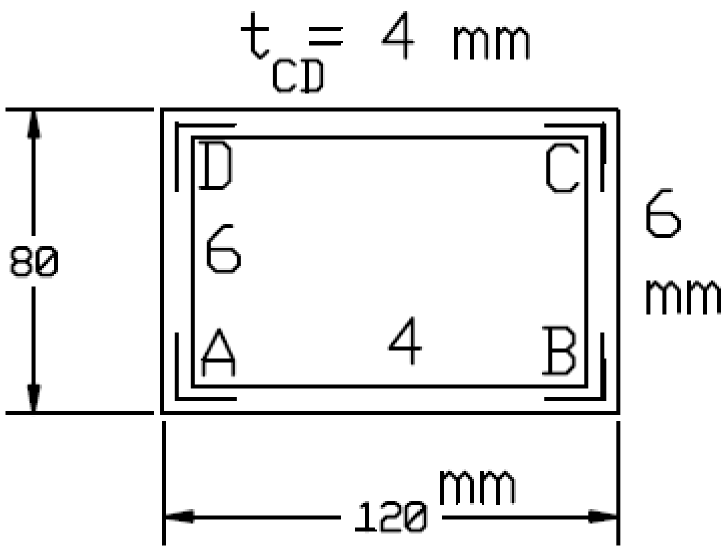


Fig. 7 (a)

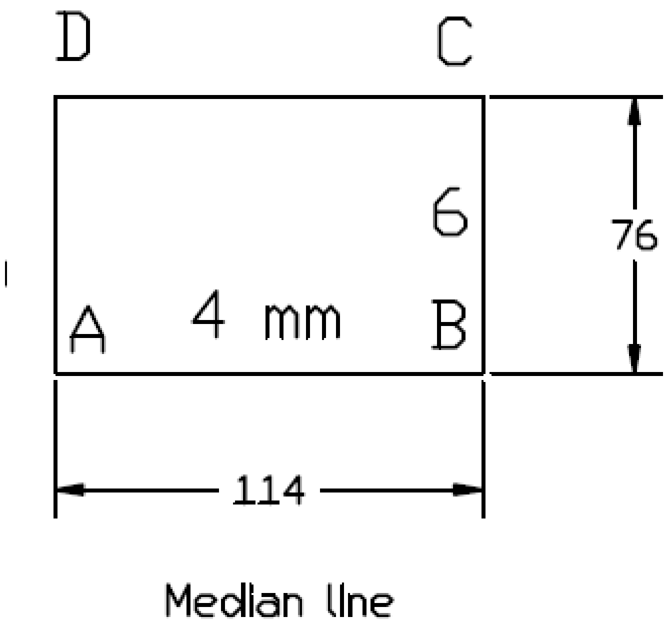


Fig. 7 (b)

Fig. 7 Example 3; closed rectangular tube.

Solution:

Calculate the dimensions of the median line, Fig. 7 (b).

AB = 120 – 6/2 – 6/2 = 114 mm

AD = 80 -4/2 – 4/2 = 76 mm

The enclosed area is

$A_0 = 0.114 \times 0.076 = 8.664 \times 10^{-3}\ m^2$

The maximum shear stress is along CD and AB where the thickness is smallest.

$$\tau_{DC} = \frac{T}{2t_{DC}A_0} = \frac{1000}{(2)(0.004)(8.664x10^{-3})} = 14.428x10^6 \quad Pa = 14.4 \quad MPa \leftarrow$$

and

$$\tau_{AB} = 14.4 \quad MPa \rightarrow$$

Calculate Φ / L

$$\oint \frac{ds}{t} = \frac{1}{4}\int_{AB} ds + \frac{1}{6}\int_{BC} ds + \frac{1}{4}\int_{CD} ds + \frac{1}{6}\int_{DA} ds = 2\left(\frac{114}{4} + \frac{76}{6}\right) = 82.333$$

$$\therefore \frac{\phi}{L} = \frac{T}{G4A_0^2}\oint \frac{ds}{t} = \frac{1000}{(80x10^9)\,(4)\,(8.664x10^{-3})^2} 82.333 = 3.428x10^{-3} \quad rad/m$$

Example 4

The steel tube has an outer radius of 25 mm and an inner radius of 20 mm, Fig. 8 . The centre of the inner surface is at a distance of 1 mm from that of the outer surface. The tube transmits a 500 Nm torque. Determine the maximum shear stress.

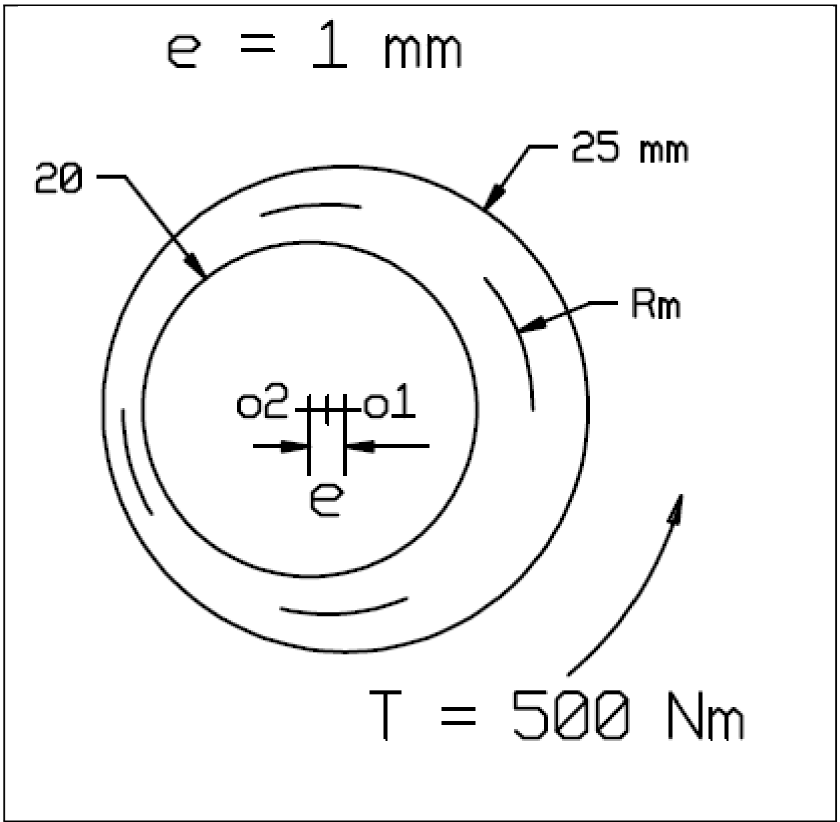


Fig. 8 Example 4; cylindrical tube with non-coincident centers. o_1 is the center of the outer surface, o_2 is that of the inner surface, and the center of the mean circle is at half the distance o_1o_2.

Solution:

The thickness is not uniform due to the offset. The minimum thickness is t_{min} = (5 – 1) = 4 mm. The enclosed area by the median line is:

A_0 = π ((25+20)/2)^2 = 1590.43 mm^2.

$$\tau_{max} = \frac{T}{2t_{min}A_0} = 39.3 \quad MPa$$

Rectangular Cross Sections

Fig. 9 (a) shows a bar with a rectangular cross section of a width “a” and thickness “b”. The maximum shear stress and the angle of twist are known to be [2,5]:

$$\tau_{max} = \frac{T}{c_1 ab^2} \qquad and \qquad \phi = \frac{TL}{Gc_2 ab^3}$$

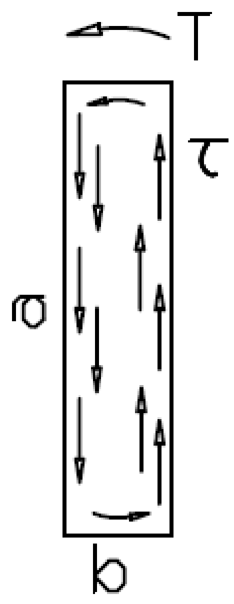


Fig. 9 (a) Direction of τ for a thin rectangular cross section (a >> b).

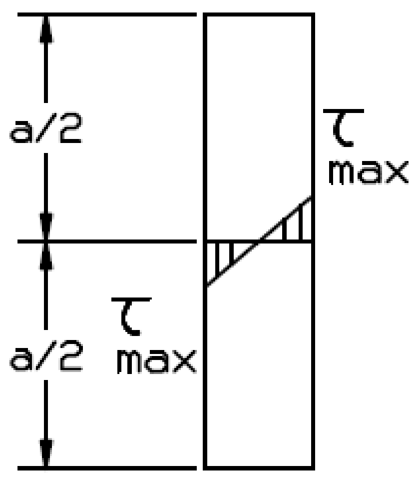


Fig. 9 (b) τ_{max} is at the middle of the long sides.

τ_{max} is at the middle of the long sides, Fig. 9 (b). The constants c_1 and c_2 are functions of a/b. Fig. 10 gives the values of these constants. The constants c_1 and c_2 equal:

$$c_1 = c_2 = \frac{1}{3}(1 - 0.63\frac{b}{a}) \quad for\ a \geq 5b$$

When a >> b, say a ≥ 20 b $c_1 = c_2$ = 1/3. It is common to assume that the torsional constants equal 1/3 even when a < 20 b.

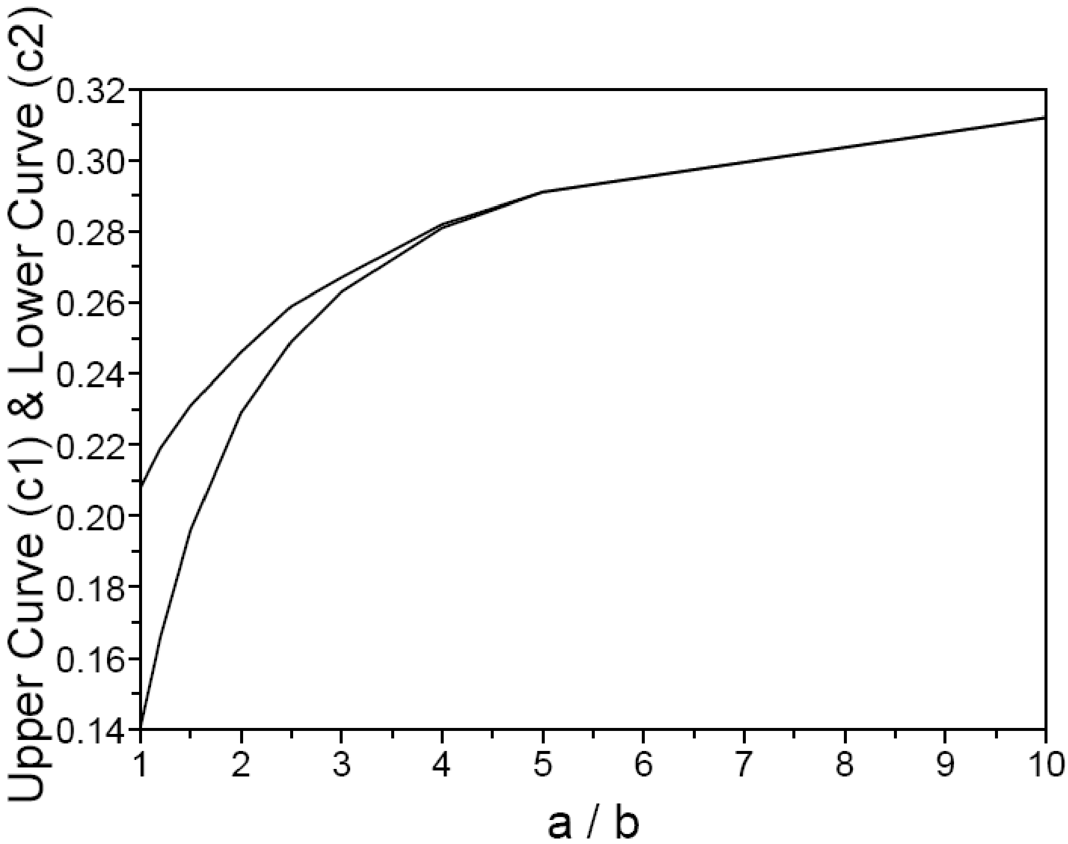


Fig. 10

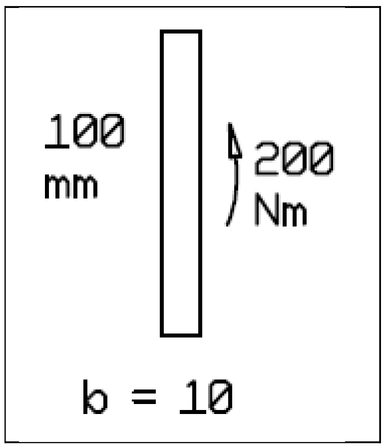
The direction of the shear stress is shown in Fig. 9 (a). The stresses are zero at the centroid and at the four corners of the cross section.

Example 5

Calculate the maximum shear stress for a bar with a rectangular cross section. The dimensions of the rectangle are a = 100 mm, and b = 10 mm. The bar transmits 200 Nm torque

- using $c_1 = ⅓$
- using $c_1 = ⅓ (1 – 0.63\ b/a)$

Solution:



For $c_1 = ⅓$

$\tau_{max-1} = T / ⅓\ a\ b^2 = 60$ MPa

Using $c_1 = ⅓ (1 – 0.63 \times 0.1) = 0.3123$, then

$\tau_{max-2} = 64$ MPa

The error is $(\tau_{max-1} - \tau_{max-2}) / \tau_{max-2} \times 100\ \% = -6.25\ \%$

Hence, using $c_1 = ⅓$ introduces an error of 6 % on the unsafe side.

Open Thin-Walled bars with Uniform Thickness

Imagine that we take any open cross section then we straighten it to a rectangular cross section, Fig. 11. The known formulas of the straightened cross section apply to the original cross section. Therefore, the maximum shear stress is

$$\tau_{max} = \frac{T}{c_1 a b^2}$$

Where "a" is the total length of the median line. The angle of twist is:

$$\varphi = \frac{TL}{G c_2 a b^3}$$

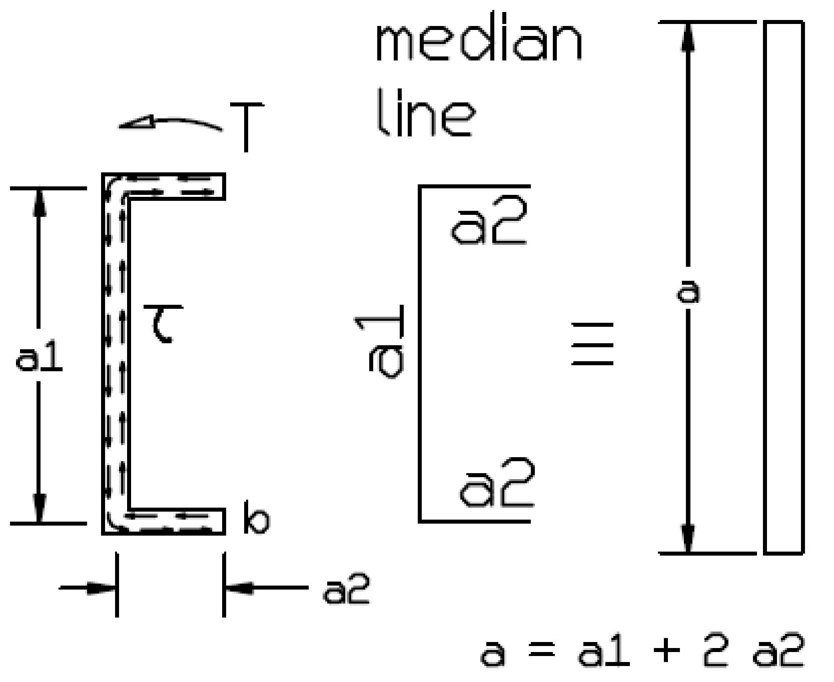


Fig. 11 The channel is equivalent to a rectangle with $a = a_1 + 2\, a_2$ and the same thickness b.

Example 6

A torque of 8 Nm is applied to the bar with the shown cross section, Fig. 12. Determine the maximum shear stress and the angle of twist per unit length. Take G = 80 GPa.

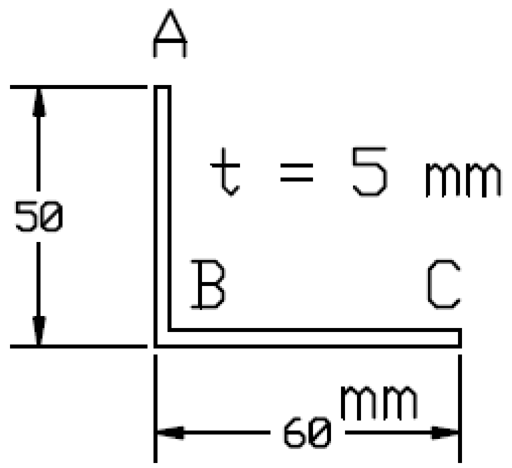


Fig. 12 (a) Example 6; an angle subjected to T = 8 Nm.

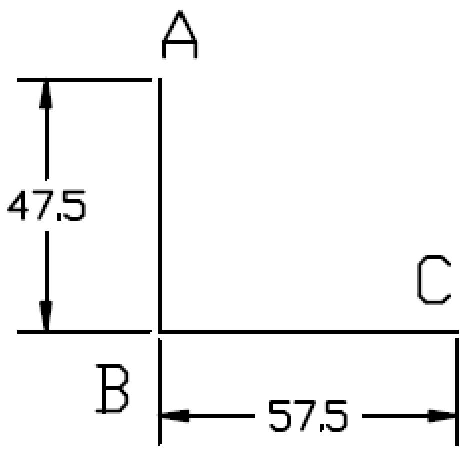


Fig. 12 (b) The median line.

Solution:

a = (50 -5/2) + (60 – 5/2) = 105 mm

b = 5 mm

Take $c_1 = c_2 = ⅓$

τ_{max} = 8 ÷ [⅓ × 0.105 × 0.005^2] = 9.1 MPa

Calculate Φ / L

Φ / L = T ÷ [G × ⅓ × a b^3] = 0.023 rad/m = 1.3 ° / m

Example 7

Calculate the maximum shear stress τ_{max} and the rate of the angle of twist Φ/L for the cylindrical tube and the open circular cross section bar , Fig. 13. Take T = 140 kNm, D_i = 230 mm, t = 30 mm, and G = 80 GPa.

Solution:

Use the exact theory for the tube

D_0 = 230 + 2 × 30 = 290 mm

J = 419.64 × 10^{-6} m^4

τ_{max} = T R_0 / J = 48.4 MPa

Φ / L = T / [GJ] = 4.170 × 10^{-3} rad / m = 0.239 ° / m

For the open cross section

b = 30 mm

a = 2 π R_m = 2 π (0.130) = 0.8168 m

τ_{max} = T / [⅓ a b^2] = 571.3 MPa (*unsafe*)

Φ / L = T /G[⅓ a b^3] = 0.238 rad / m = 13.6 ° / m

The open cross section bar is unsafe (for most materials) because the shear stresses are very high. In the elastic range

$$\frac{\tau_{open}}{\tau_{tube}} = 11.8 \quad and \quad \frac{\phi_{open}}{\phi_{tube}} = 57$$

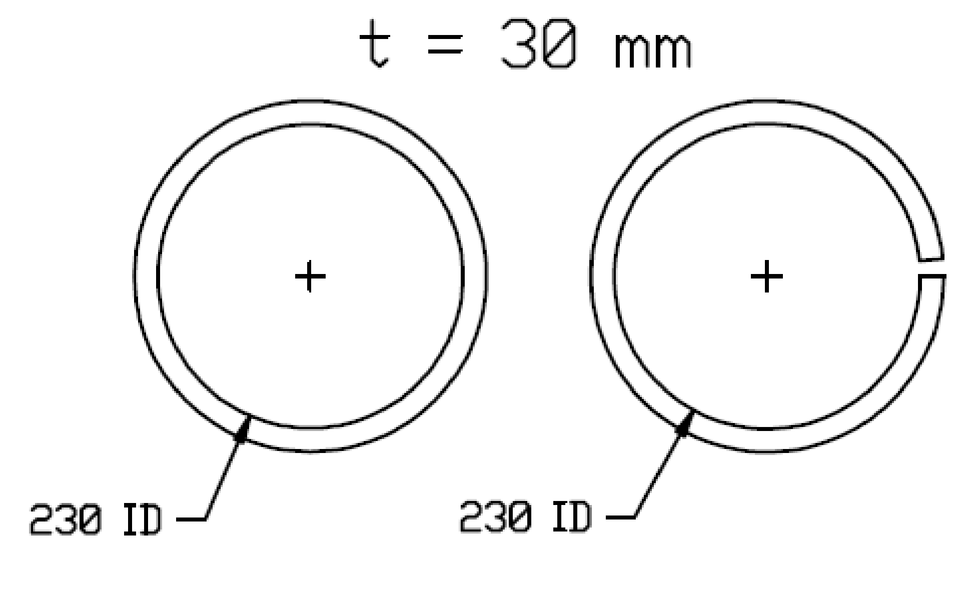


Fig. 13 Example 7; comparison between closed and open cross sections. The gap is of zero length.

Branched and Tapered Open Cross Sections

Fig. 14 shows examples of open cross sections that are composed of rectangular segments. The angle of twist for a cross section made of n rectangular segments is

$$\phi = \frac{TL}{GK} \quad ; \quad where \quad K = \sum_{i}^{n} \frac{1}{3} a_i b_i^3$$

Which will be proved later for the configuration shown in Fig. 15. In addition, the shear stress at any segment "i" is

$$\tau_i = \frac{Tb_i}{K}$$

Hence, the maximum shear stress is at the most thick segment

$$\tau_{\max} = \frac{Tb_{\max}}{K}$$

Proof

Take a cross section made of three segments, Fig. 15. Each segment has its own thickness b_i. The applied torque is distributed among the three segments. Hence,

$T = T_1 + T_2 + T_3$

Each segment rotates by the same angle of twist Φ.

$\therefore \Phi = \Phi_1 = \Phi_2 = \Phi_3$

$$\therefore \frac{T_1 L}{K_1 G} = \frac{T_2 L}{K_2 G} = \frac{T_3 L}{K_3 G}$$

Where, L is the length of the bar.

$$\therefore \frac{T_1}{K_1} = \frac{T_2}{K_2} = \frac{T_3}{K_3} = \frac{T_1 + T_2 + T_3}{K_1 + K_2 + K_3} = \frac{T}{K}$$

Therefore, the torsion constant of the cross section

$K = K_1 + K_2 + K_3$

Now,

$$T_1 = \frac{TK_1}{K}$$

$$\therefore \phi = \phi_1 = \frac{T_1}{K_1}\frac{L}{G} = \frac{T}{K}\frac{L}{G}$$

and

$$\tau_1 = \frac{T_1}{\frac{1}{3}a_1 b_1^2} = \frac{TJ_1}{K}\frac{1}{\frac{1}{3}a_1 b_1^2} = \frac{Tb_1}{K}$$

$$\tau_2 = \frac{Tb_2}{K} \quad and \quad \tau_3 = \frac{Tb_3}{K}$$

The maximum shear stress develops at the segment with the largest thickness

$$\tau_{max} = \frac{Tb_{max}}{K}$$

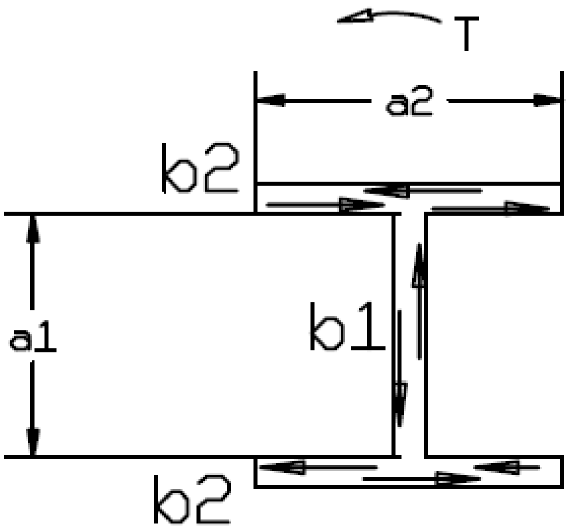


Fig. 14 (a)

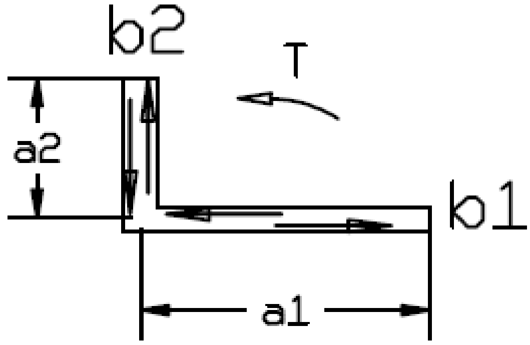


Fig. 14 (b)

Fig. 14 Open cross sections with variable thickness.

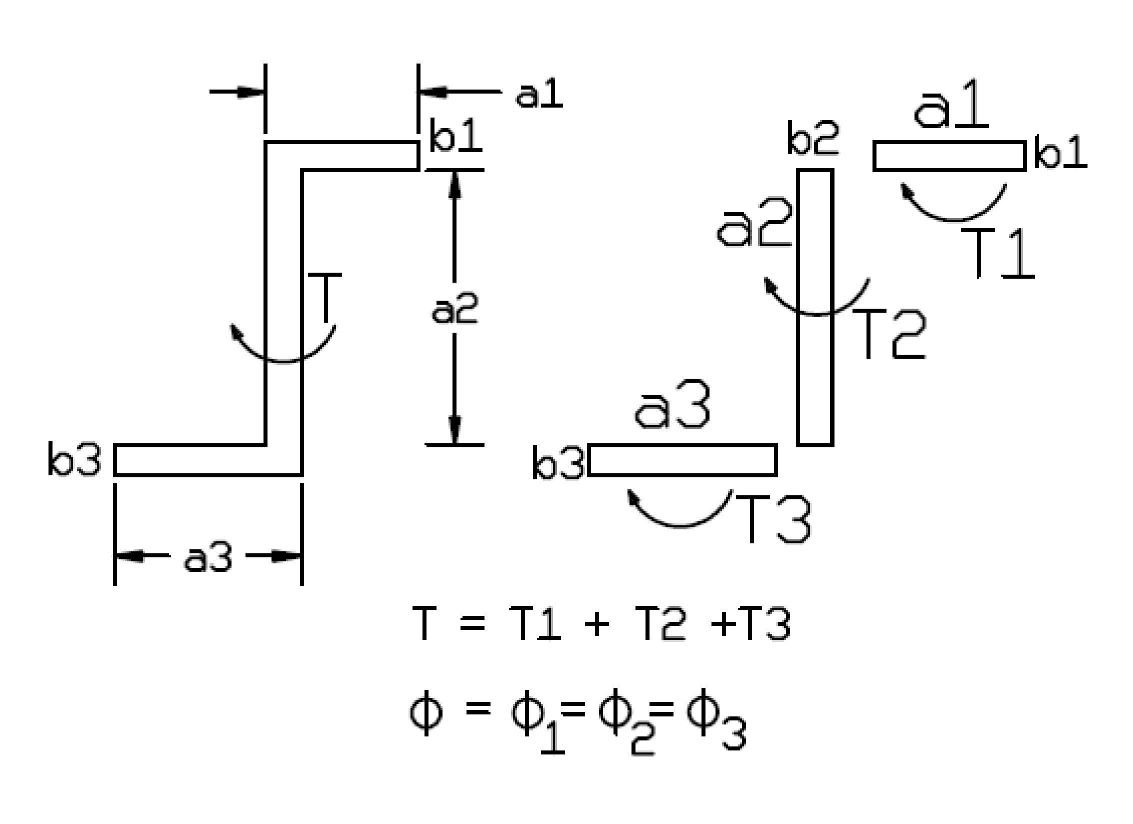


Fig. 15 Proof of K = ∑ K_i, $\tau_i = \frac{Tb_i}{K}$, and $\tau_{max} = \frac{Tb_{max}}{K}$

Tapered Cross Sections

Fig. 16 shows a tapered cross section. We can divide the cross section into n segments. Each segment has a thickness of b_i and length a_i.

$\therefore K = \sum \frac{1}{3} a_i b_i^3$

Better

$K = \int \frac{1}{3} b_i^3 \, dx$

and

$$\tau_{max} = \frac{Tb_{max}}{K} \qquad and \qquad \phi = \frac{TL}{GK}$$

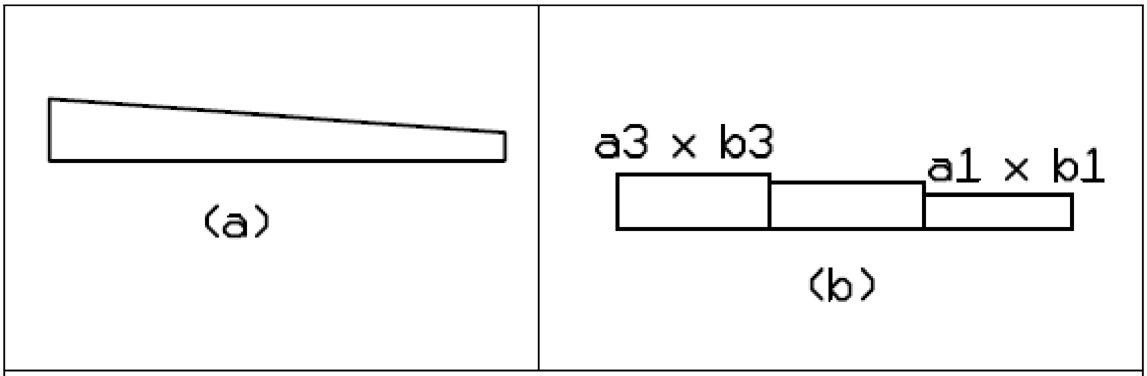


Fig. 16 (a) Tapered cross section. (b) The section divided to segments.

Example 8

A steel I beam is subjected to a torque of 5000 Nm, Fig. 17.

- Determine the maximum shear stress τ_{max} and its location.
- Determine the angle of twist per unit length. Neglect stress concentrations. Take $c_1 = c_2 = ⅓$ for the flanges and the web. G = 80 GPa.

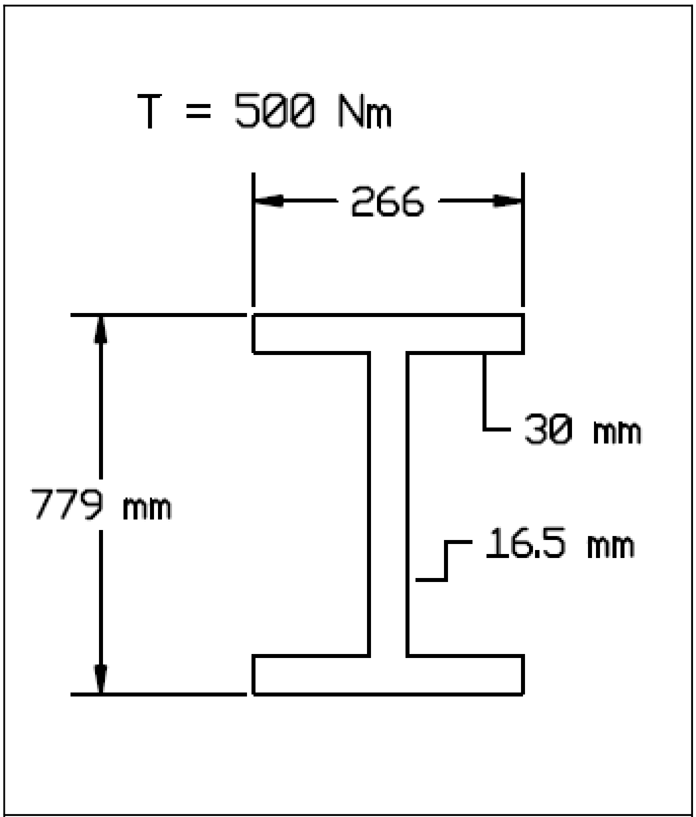


Fig. 17 Example 8; an I beam.

Solution:

The torsion constant:

$K = ⅓ [266 \times 30^3 + 266 \times 30^3 + (779\text{-}60) \times 16.5^3] = 5.865 \times 10^6 \text{ mm}^4 = 5.865 \times 10^{-6} \text{ m}^4$

$$\tau_{max} = \frac{Tt_{max}}{K} = \frac{5000\ (0.03)}{5.865\,10^{-6}} = 25.58 \quad MPa$$

The maximum shear stress is at the mid point of the upper surface of the upper flange and at the corresponding point at the other flange.

Calculate the rate of the angle of twist

$$\frac{\phi}{L} = \frac{T}{GK} = 0.011 \quad rad/m = 0.611 \quad ^{\circ}/m$$

Example 9

A hollow tube with radial fins is subjected to torque T = 2 kNm, Fig. 18 (a). Find the torque transmitted to the fins and the maximum shear stress. Use the correct values of c_1 and c_2.

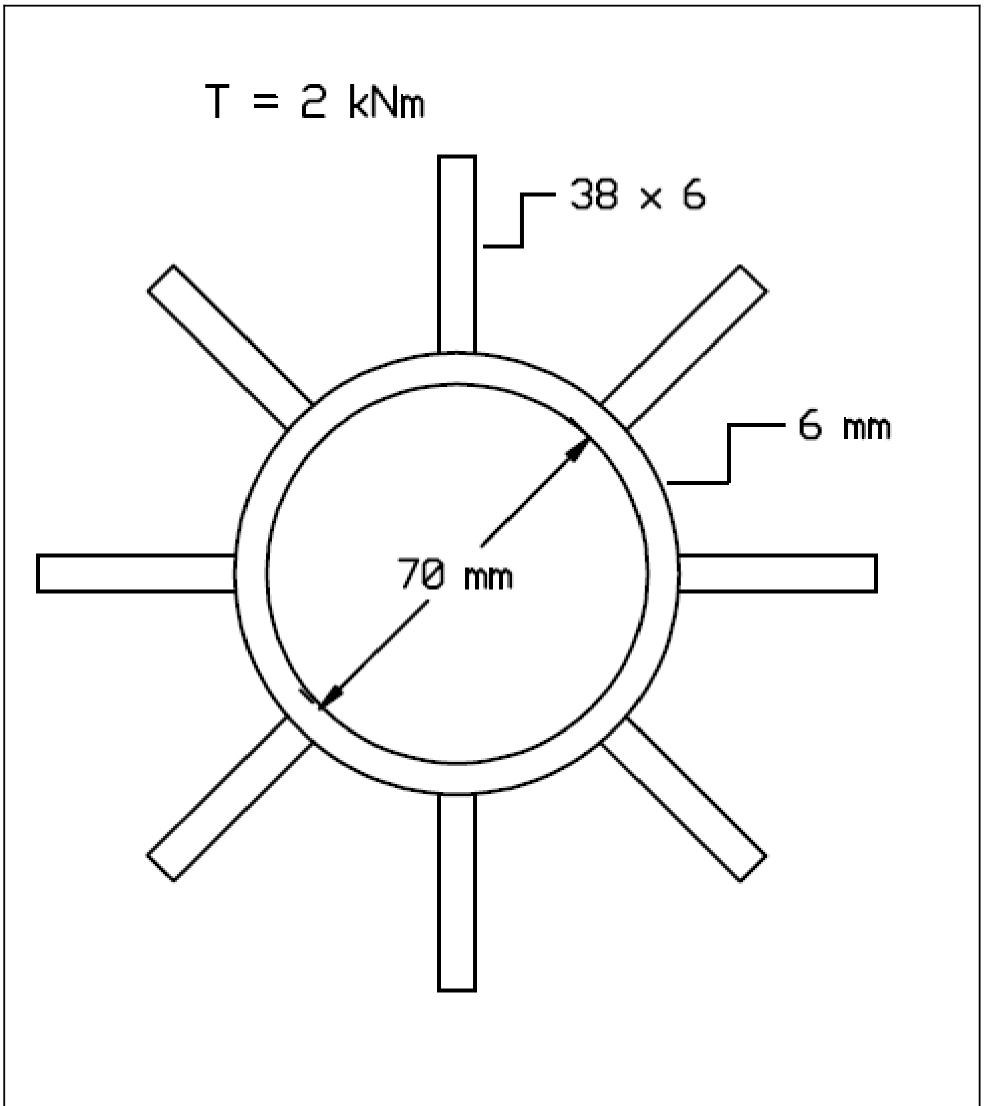


Fig. 18 (a) Example 9, a mixer.

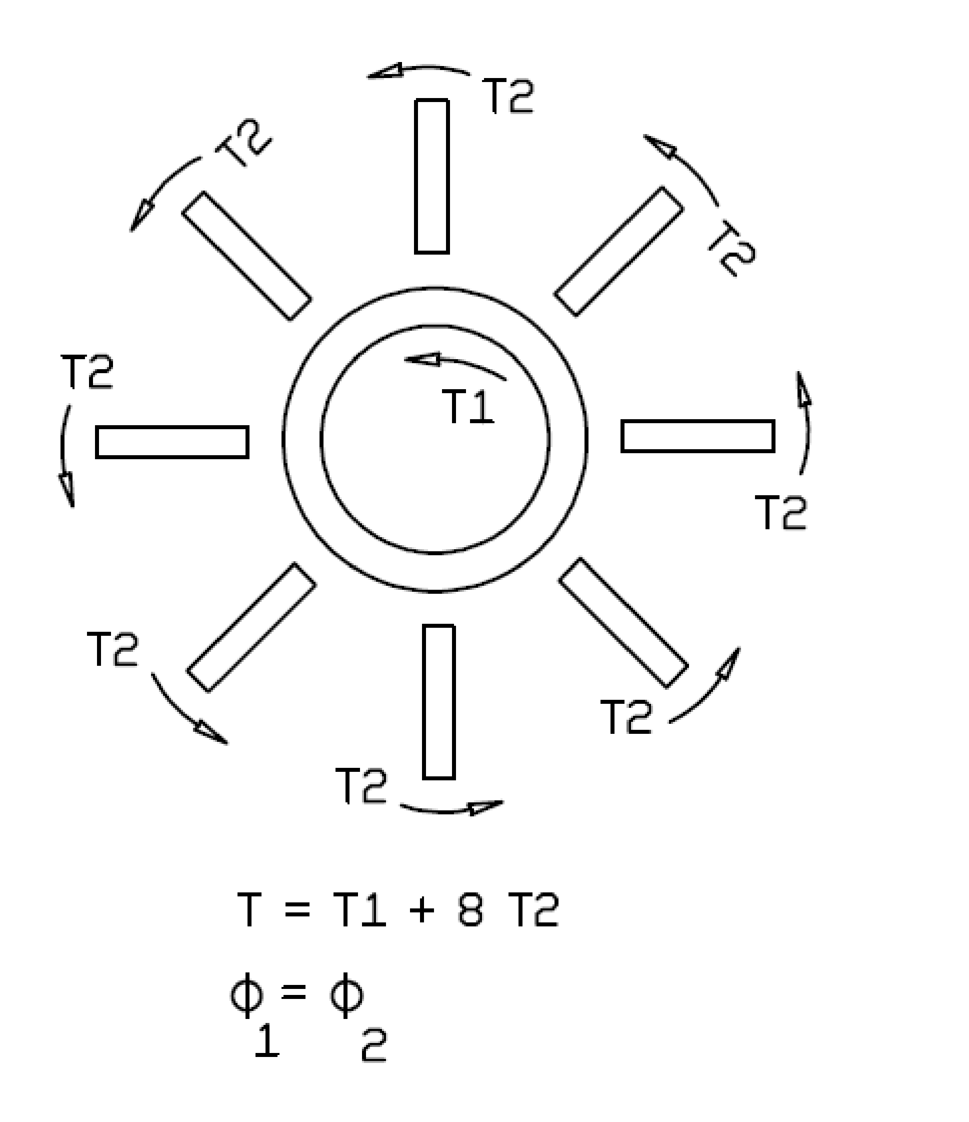


Fig. 18 (b) $\phi = \phi_1 = \phi_2$. Each fin transmits a torque T_2, and the tube transits T_1 .

Solution:

Each fin has a rectangular cross section, Fig. 18 (b).

$c_1 = c_2 = \frac{1}{3}(1 - 0.63 \times 6 / 38) = 0.3$

The tube has $K_1 = J_1$.

$T = T_1 + 8\ T_2$

$$\phi = \frac{T_1 L}{J_1 G} = \frac{T_2 L}{K_2 G} \qquad \therefore \frac{T_1}{J_1} = \frac{T_2}{K_2} = \frac{T_1 + 8T_2}{J_1 + 8K_2} = \frac{T}{J_1 + 8K_2}$$

$K_2 = 0.3 \times 0.038 \times 0.006^3 = 2.4624 \times 10^{-9}\ \text{m}^4$

$J_1 = \pi / 32\ [0.082^4 - 0.070^4] = 2.08152 \times 10^{-6}\ \text{m}^4$

The torsion constant K for the cross section

$K = J_1 + 8\ K_2 = 2.1012 \times 10^{-6}\ \text{m}^4$

The twisting moment carried by each fin

$$T_2 = \frac{K_2}{J_1 + 8K_2} T = \frac{2.4624 x 10^{-9}}{2.1012 x 10^{-6}} 2000 = 2.34 \quad N.m$$

The 8 fins take a twisting moment of 8 × 2.34 = 18.75 Nm

The twisting moment carried by the tube

$$T_1 = \frac{J_1}{K} T = \frac{2.08152 x 10^{-6}}{2.1012 x 10^{-6}} 2000 = 1981.25 \quad N.m$$

The fins carry only 0.94 % of the applied torque.

The maximum shear in the fins is

$$\tau_{\max-2} = \frac{T_2}{c_1 a b^2} = 5.7 \quad MPa$$

Moreover the maximum shear stress in the tube is

$$\tau_{\max-1} = \frac{T_1 R_0}{J_1} = \frac{1981.25\,(0.035 + 0.006)}{2.08152\, x 10^{-6}} = 39 \quad MPa$$

Hence, the maximum shear stress is 39 MPa.

Rolled Steel Cross Sections

Rolled steel sections contain fillets and tapered segments. Handbooks[2] list formulas for calculating the torsional constants, and national organizations (such as The American Institute of Steel Construction) publish explicit values of K.

Stress Concentration

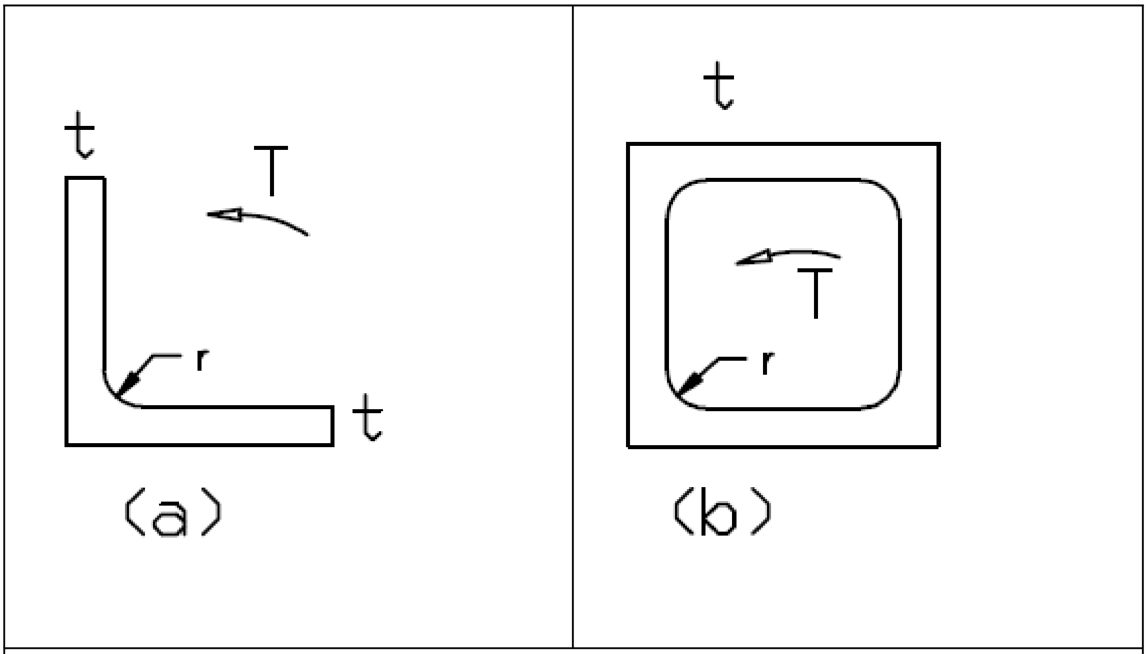


Fig. 19 Stress concentration factor.

[2] W. C. Young, "Roark's Formulas for Stress and Strain", sixth edition, McGraw-Hill, 1989.

The stresses at the neighbourhood of the inner corners (Figs. 19 (a) and (b)) attain high-localized values due to the sudden change in geometry. These peak stresses equals

τ_{peak} = (SCF) τ_{max}

Where, the stress concentration factor (SCF) depends on the fillet radius r. Generally, the (SCF) decreases with the increase of "r"[3]. The following table gives some values of the (SCF)[4].

r / t	Fig. 19 (a); an angle section	Fig. 19 (b); a box tube
0.25	2	2.5
1	1.56	1.25
1.5	1.6	1.08

Hence, it is recommended to use a fillet of radius r = t.

The Displacements of Open Cross Sections

Displacements in the Plane y-z of the Cross Section

The cross section rotates without distortion by the twist angle Φ about a fixed point known as the torsional centre (TC). Fig. 20 shows the angle of twist for two cross sections. Because the I beam possesses two axes of symmetry, its (TC) is at the point of intersection of these axes. The channel has one axis of symmetry and the (TC) is along this axis. The exact location of the (TC) is to be determined. The (TC) coincides with another point named the shear centre. The shear centre and its location will be covered later in the course.

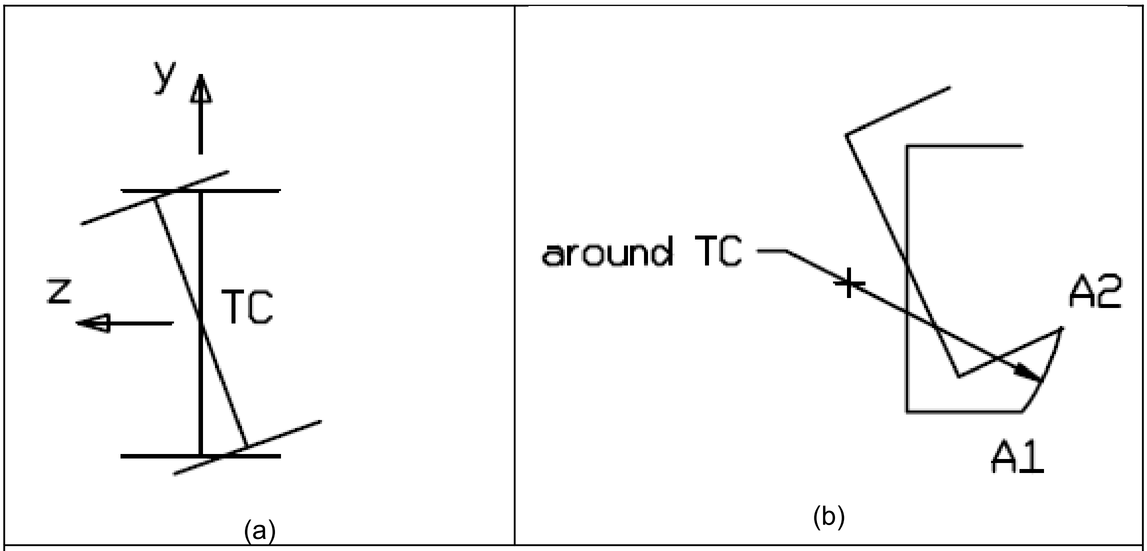


Fig. 20 The torsional center (center of rotation) for:
(a) an I beam is located at the intersection of the axes of symmetry y and z, and for
(b) a channel is located along the horizontal axis of symmetry (marked by a "+").

[3] However, for torsion of an open cross section increasing r excessively results in increasing the thickness. This causes the stresses to increase after a certain r/t value. Hence, for the angle section r = t is recommended.

[4] J.H. Huth, "Torsional Stress Concentration in Angle and Square Tube Fillets", Journal of Applied Mechanics, ASME, Vol 17, No 4, 1950, pp. 388-390.

The Axial Displacement u

The twisting moment when acts on most thin-walled open cross sections produces an axial displacement u = u(y, z). The displacement u varies from one point to another. Hence, the plane of the cross section deforms to a wavy shape. For instance, Fig. 21 shows the axial displacement of a rectangular open cross section.

When the end of the bar is fixed to another member, this prevents the axial displacements of the points of this end. This has two outcomes:

- It reduces the angle of twist Φ, which is usually desirable.
- It increases the stresses in the zone of the restrained plane.

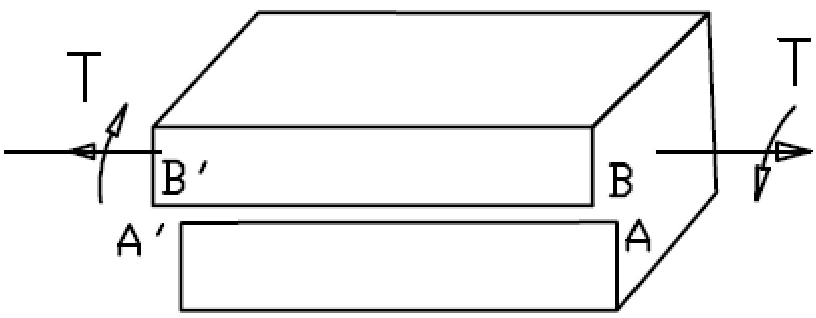


Fig. 21 The axial displacement of a rectangular cross section. The edge A'A moves relative to B'B. Hence, the plane of the cross section distorts.

Field of Application

Frames of machines and vehicles are made up of thin-walled members. Improving their torsional rigidity and strength is important. Simplified and practical analyses are a good step toward achieving this improved response. Blodgett[5] presents practical approaches for the analysis of frames and bases of machines. Finite element programs provide assistance in evaluating any proposed frame configuration.

Conclusion

The torsional rigidity and strength of closed thin-walled members are much better than open ones. Moreover, their formulas are more accurate and less sensitive to boundary conditions.

References

1. S. E. Bayoumi, Mechanics of Deformable Solids, Cairo University, 1971.
2. F. P. Beer, Johnston Jr, and J.T. DeWolf, Mechanics of Materials, 4th ed. (SI units), McGraw-Hill, New York, 2006.
3. A.P. Boresi, and R.J. Schmidt, Advanced Mechanics of Materials, 6th ed., John Wiley, New York, 2003.

[5] O. W. Blodgett, "Design of Weldments", The James F. Lincoln Arc Welding Foundation, 1963 (still being printed). www.weldinginnovation.com

4. R. D. Cook, and W. C. Young, Advanced Mechanics of Materials, Macmillan Publishing Company, New York, 1985. (2nd ed. 1999.)
5. R. C. Hibbeler, Mechanics of Materials, SI 2nd ed., Prentice Hall (Pearson Education), Singapore, 2005.

Appendix I Shear Flow for Tubular Cross Sections

The shear stress at any point in the plane of the cross section is associated with an equal shear stress along the axial direction, Fig. A1.

For the free body diagram of portion ab

$\sum F_x = 0$

$\therefore \tau\ dA\ )_a - \tau\ dA\ )_b = 0$

but $dA\ )_a = t_a\ dx$

and $dA\ )_b = t_b\ dx$

$\therefore \tau t\ )_A = \tau t\ )_B = q =$ constant

Where, q is the shear flow.

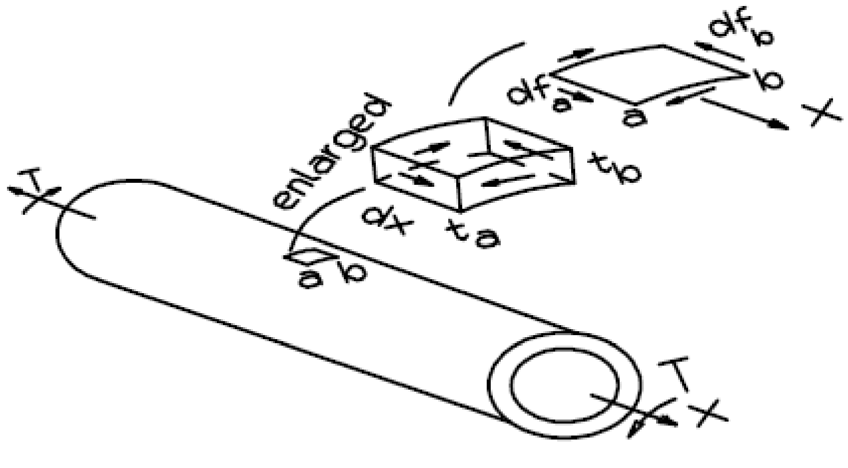


Fig. A1 $q = \tau\ t\ )_a = \tau\ t\ )_b =$ constant. The idea of the proof is $\sum dF = 0$, along the axial direction. Note: The cross section has a general shape with variable thin thickness.

Appendix II Shear Stress Bredt's Formula

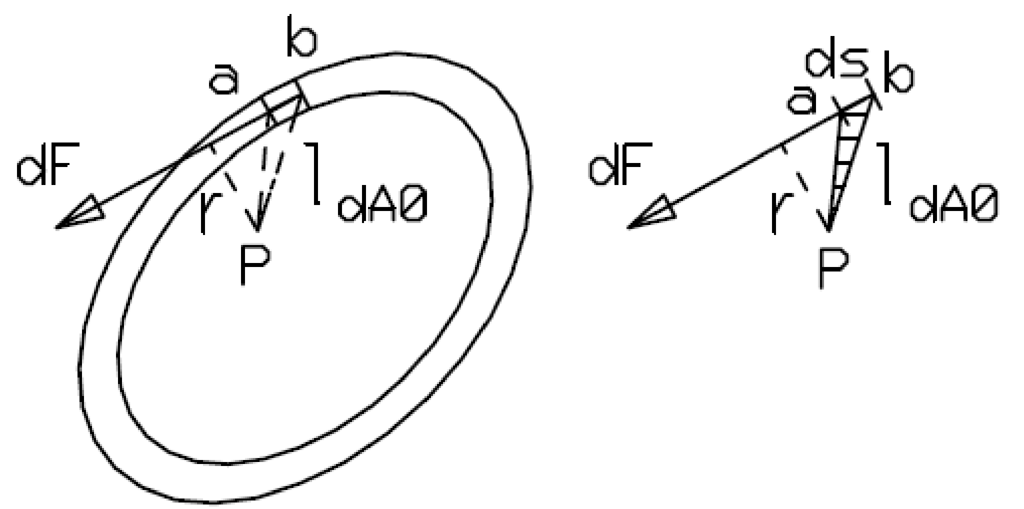


Fig. A2 Proof of $\tau_i = T / (2\ t_i\ A_0)$. Point P is arbitrarily located in the plane.

At any point in the cross section, take a small area across the thickness of length ds. The shear stress acting at this area produces an increment of force dF, Fig. A2.

$dF = \tau\ t\ ds = q\ ds$

This force develops an incremental torque dT about a typical point P.

$dT = dF\ r = q\ ds\ r = q\ r\ ds$

Where r is perpendicular to the line ds. The area of the triangle of base ds and vertex P is dA_0.

$dA_0 = \frac{1}{2}\ ds\ r$

$\therefore dT = 2\ q\ dA_0$

Integrate dT along the closed path of the median line to get T.

$$T = \oint dT = 2q\oint dA_0 = 2qA_0$$

$$\therefore\ q = \frac{T}{2A_0}$$

$$\therefore\ \tau = \frac{T}{2tA_0}$$